THE THEORY ON NEUTRON-PROTON SCATTERING

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ABSTRACT. The Wavestatistical Theory of Neutron-Proton Scattering is developed using an interaction potential of Yukawa type, *viz.*, $V = \frac{\Lambda}{r} e^{-ar}$. The theoretical formula for the intensity of scattering is found to agree with the experiment of Harkins and others for $\sqrt{\Lambda} = 6e$ and $a = .2848 \times 10^{13}$. This value of a gives 110 *e.m.* as the mass of mesotron taking part in the exchange in neutron-proton interaction. The mass so determined is exactly the same as that obtained from the binding energy of deuteron and from proton-proton scattering.

In the theory of neutron-proton scattering recently developed by one of the authors (Kar and Basu, 1939) the interaction potential has been taken in the form $V = -V_0 e^{-ar}$. The formula derived has been found to agree only qualitatively with the experiments of Chadwick (1933) and others. In the present paper it is proposed to develop the theory of scattering somewhat on similar lines, on the basis of an interaction potential suggested by Yukawa.

The wave equations of the incident neutron inside and outside the potential field of the proton, the motion being referred to C-system, are

$$\Delta\chi + k^2\left(1 - \frac{V}{E}\right)\chi = 0 \qquad \ldots \quad (1)$$

and

$$\Delta\chi_0 + k^2\chi_0 = 0 \qquad \ldots \quad (1.1)$$

where $k = \frac{\pi M v}{h}$ and $E = \frac{1}{4}Mv^2$. Proceeding in the usual manner (Kar, *et al* 1937), we have for the first order scattering function

$$\lambda_1\chi_1(r_2) = -\frac{1}{4\pi} \cdot \frac{k^2}{E}\int V(r_1)\chi_0(r_1) \cdot \frac{e^{ikr_{12}}}{r_{12}} dr_1 \qquad \ldots \quad (2)$$

where

$$\chi_0 = \frac{1}{\sqrt{v}} e^{ikx_1}, \qquad \ldots \quad (2.1)$$

the incident wave being supposed to move along the x-axis. On integrating (2) as before (Kar, 1937), we have

$$\lambda_1\chi_1(r_2) = -\frac{2k \operatorname{cosec} \frac{1}{2}\psi}{Mv^2} \cdot \frac{e^{ikr_2}}{\sqrt{v}\, r_2} \int_{r_0}^{\infty} \sin k'r\, V(r)\, r\, dr \qquad \ldots \quad (2.2)$$

where $$k' = 2k \sin \tfrac{1}{2}\phi.$$

Eq (2.2) may be written in the simple form

$$\lambda_1\chi_1(r) = \frac{\operatorname{cosec}^2 \frac{1}{2}\phi}{Mv^2} \cdot \frac{e^{ikr}}{\sqrt{v_1}\, r} F(r_0) \qquad \ldots \quad (2.3)$$

where $$F(r_0) = -k' \int_{r_0}^{\infty} \sin k'r V(r) r\, dr \qquad \ldots \quad (2.4)$$

Proceeding in the usual way it may be easily shown that the critical approach r_0, which must be always positive, is given, for an attractive force of interaction, by the formula (Kar, 1942)

$$r_0 = \frac{\operatorname{cosec}^2 \frac{1}{2}\phi}{Mv^2} \cdot 2\rho(\sin \tfrac{1}{2}\phi - \sin^2 \tfrac{1}{2}\phi) G(r_0) \qquad \ldots \quad (3)$$

where $$G(r_0) = -k' \int_{r_0}^{\infty} \sin k'(r - r_0) V(r) r\, dr \qquad \ldots \quad (3.1)$$

Now, taking the interaction potential to be of Yukawa type, *viz.*, $V(r) = -\frac{A}{r} e^{-\alpha r}$, we find the values of $F(r_0)$ and $G(r_0)$. On substituting these values in (2.3) and (3) we get for the scattering function and the critical approach

$$\lambda_1\chi_1 = \frac{4\pi^2 MA}{h^2} \cdot \frac{e^{-\alpha r_0}\left\{\cos k'r_0 + \frac{\alpha}{k'} \sin k'r_0\right\}}{\alpha^2 + k'^2} \cdot \frac{e^{ikr}}{\sqrt{v}\, r} \qquad \ldots \quad (4)$$

$$r_0 = 2.7 \times \frac{4\pi^2 MA}{h^2} \cdot \frac{e^{-\alpha r_0}}{\alpha^2 + k'^2} (\sin \tfrac{1}{2}\phi - \sin^2 \tfrac{1}{2}\phi) \qquad \ldots \quad (4.1)$$

If θ is the angle of scattering in the laboratory system, we have $2\theta = \phi$. Thus we have for the relative intensity of scattering and the critical approach in L-system

$$I = g^2 f^2 \, 4 \cos\theta \, . \, 2\pi \sin\theta d\theta \qquad \ldots \quad (5)$$

and $$r_0 = 2.7 \times (\sin\theta - \sin^2\theta) g \qquad \ldots \quad (5.1)$$

where $$g = \frac{4\pi^2 MA}{h^2} \cdot \frac{e^{-\alpha r_0}}{\alpha^2 + k'^2} \qquad \ldots \quad (5.2)$$

and $$f = \cos k'r_0 + \frac{\alpha}{k'} \sin k'r_0 \qquad \ldots \quad (5.3)$$

In order to verify our formula (5) quantitatively we have to depend on the only experiment done so far by Harkins, Kamen, Newson and Gans (1936). Their experimental values are, however, given for angular ranges of ten degrees and so it is necessary to integrate (5) between θ_1 and θ_2. Because the angular range is small we may take g and f outside the sign of integration giving them their mean values, *viz.*, g_m and f_m. We have then from (5) after integration

$$I(\theta_1, \theta_2) = 2\pi g_m^2 f_m^2 (\cos 2\theta_1 - \cos 2\theta_2) \qquad (5.4)$$

where

$$g_m = \frac{4\pi^2 \mathrm{MA}}{h^2} \frac{r_{0m}}{\alpha^2 + k'^2_m} \tag{5.5}$$

$$f_m = \cos k'_m r_{0m} + \frac{\alpha}{k'_m} \sin k'_m r_{0m} \tag{5.6}$$

and

$$r_{0m} = 2.7 \times (\sin \theta_m - \sin^2 \theta_m) g_m \quad \ldots \tag{5.7}$$

Again, because the absolute values of the intensity are not given in the experiment cited above, formula (5.4) cannot be verified directly. However, it is obvious that the experimental values which are proportional to the absolute values, are given by the formula

$$\mathrm{I}(\theta_1, \theta_2) = \mathrm{B} \times 2\pi g_m^2 f_m^2 (\cos 2\theta_1 - \cos 2\theta_2) \quad \ldots \tag{5.8}$$

where B is the unknown constant of proportion. On using tentatively the values, *viz.*, $\sqrt{\mathrm{A}} = 6e$ already determined from proton-proton scattering (Kar, 1942) and $\alpha = .2848 \times 10^{13}$ obtained in our previous paper (Kar and Roy, 1943), we easily find the unknown constant of proportion B from one experimental value at a given angular range. Having got the value of B once for all we are in a position to calculate the values of the intensity of scattering at any other angular range. The theoretical values so obtained from (5.8) and the experimental values are given respectively by the continuous and the dotted curves in Fig. 1.

It is apparent from Fig. 1 that the theoretical curve has a peak at 45°, showing that in C-system the scattering is isotropic. This is confirmed by the experiments of Chadwick (1933) and others. The peak of the experimental curve is, however, at 65°. It it is evident that if this difference is neglected, the agreement it as is should be. Now it has been pointed out by Dee and Gilbert (1937), that the shift of the maximum intensity towards large angle, as observed by Harkins and others is due to inhomogeneity of the incident beam of neutrons from the source taken by them.

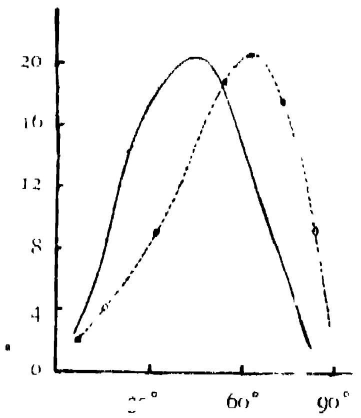

Fig. 1

It should be noted that the values of Λ and a for which the theoretical curve is in agreement with the experiment are very nearly the same as those obtained from proton-proton scattering (Kar 1942), and also same as the value of α obtained from the binding energy of deuteron (Kar and Roy, 1943). This shows that the nature of the short range force is essentially the same in neutron-proton and proton-proton interactions. Now, because α has the same value in the different theories, the mass of mesotron which is responsible for the short range force in neutron and proton, is also same. And it has been found to be 110 electron unit from the usual formula

If, now, formula (5) for the intensity of scattering for the elementary solid angle $2\pi \sin \theta d\theta$ be integrated over the limits o and $\pi/2$, we get the total scattering cross-section σ. Logarithms of the cross-sections thus determined for different velocities are plotted against the logarithms of incident energy. And the curve is given in Fig. 2

The circles indicate the experimental values. It is obvious that there is very good agreement for thermal neutrons having low velocity. However, for higher velocities there are some departures.

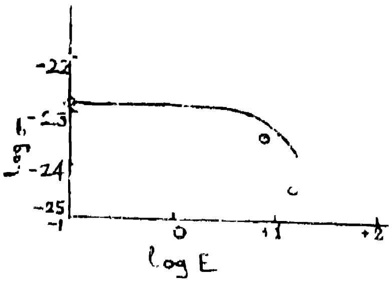

FIG. 2

Before concluding, we shall make a few remarks on some of the special features of the wavestatistical theories of scattering so far developed (Kar, *et al*, 1942), for different kinds of interactions. The fundamental assumption made in the above theories, is that the incident particle approaches the scattering particle only up to a certain distance r_0 which is called the 'critical approach.' It has nothing to do with the size of the nucleus and, in fact, it is much greater than the size. It has been shown that the critical approach depends on the velocity of the incident particle and also on the angle of scattering. It is obvious that it decreases with the increase of the incident velocity for a given angle of scattering, and also for a given incident velocity it decreases with the increase of scattering angle. Now, the volume round the scatterer which is unperturbed by the incident particle due to the critical approach may be called the 'excluded volume.' This volume has evidently no contribution to the total intensity of scattering. Now, in deriving Born-Rutherford, Holtzmark and other wave-mechanical formulæ of scattering the above excluded volume has been wholly neglected. In other words, the scattering by the excluded volume is taken into consideration. As a result their theoretical values are always much higher than the experimental values. It is well known (Mott and Massey, 1933), that at high velocity of incidence and large angles of scattering, Born-Rutherford formula is found to agree fairly well with the experiment. At this region the excluded volume is obviously negligible. Thus we find that the wave mechanical formulæ generally give much higher values, whereas the wavestatistical formulæ are decidedly in better agreement with the experiment. And in some cases the agreement is found to be almost exact.

Apart from the experimental evidences which confirm the wavestatistical theories, there are theoretical justifications in support of the fundamental assumption of critical approach. The incident beam of particles moving in straight line in a given direction is deflected due to the presence of the scatterer. Thus the scatterer acts as a source of perturbation and imposes some boundary conditions on the motion of the incident beam of particles. On account of these boundary conditions, the incident beam approaches the scatter only upto a certain limit. If the wave character of the particles is completely disregarded, the critical approach is evidently given for Coulomb field by the distance of the vertex of the hyperbolic path from the centre of the scattering particle. However, the wavestatistical character of the particles constituting the incident beam cannot be ignored. Accordingly, in the different wavestatistical theories of scattering the critical approach r_0 for any potential field is always determined by the wavestatistical method using the boundary conditions at $r = r_0$

$$\left.\begin{aligned} \chi_0 + \lambda_1 \chi_1 &= 0, \\ \frac{d\chi_0}{dr} + \lambda_1 \frac{d\chi_1}{dr} &, \end{aligned}\right\} \qquad \ldots \quad (6)$$

It is interesting to note that for Coulomb field the wavestatistical method gives exactly the same value of the critical approach as the dynamical method except an additional numerical factor 1.35, which is known as the dynamical defect factor.

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